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COLLEGE OF BASIC AND APPLIED SCIENCES
DEPARMENT OF MATHEMATICS AND PHYSICAL SCIENCES

Course Code: PHY 111 **Course Title:** GENERAL PHYSICS I

Lecture Notes B3 – Gas Laws

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Content:

Introduction

When a gas is subjected to heat, its behaviour can be described by pressure (P), volume (V), temperature (T) and the number of moles (n) of the gases. These four variables or parameters are used to describe the state of a given mass of a gas. Firstly in this topic, we shall discuss the relationship between the temperature, pressure and volume of a gas and thereafter examine the behaviour of gases using these parameters to deduce the various gas laws.

Objectives

At the end of this topic, you should be able to:

1. State the different gas laws
2. Explain the gas laws through the use of graphs
3. Distinguish between a real gas and an ideal gas
4. Express the equation of state of an ideal gas
5. Solve problems on these gas laws.

Boyle’s Law

Boyle’s law describes the behaviour of a given mass of gas by keeping are kept temperature, T and the number of mole, n of a gas constant and then subjecting pressure, P and volume, V to change.

According to Robert Boyle in 1662, Boyle's law states that the pressure (P) on a given mass of gas is inversely proportional to its volume (V) provided its temperature is kept constant.

$$P \propto \frac{1}{V} \qquad (1)$$

$$\therefore \quad P = \frac{K}{V}$$ where, K is a constant of proportionality.

$$\therefore \quad PV = K = \text{Constant} \qquad (2)$$

$$P_1V_1 = P_2V_2 = K \qquad (3)$$

Boyle's law is applied in air compressors and exhaust (vacuum) pumps.

If you plot a graph P versus $\frac{1}{V}$, then the graph would be as given below in fig. 1(i) and fig. 1(ii).

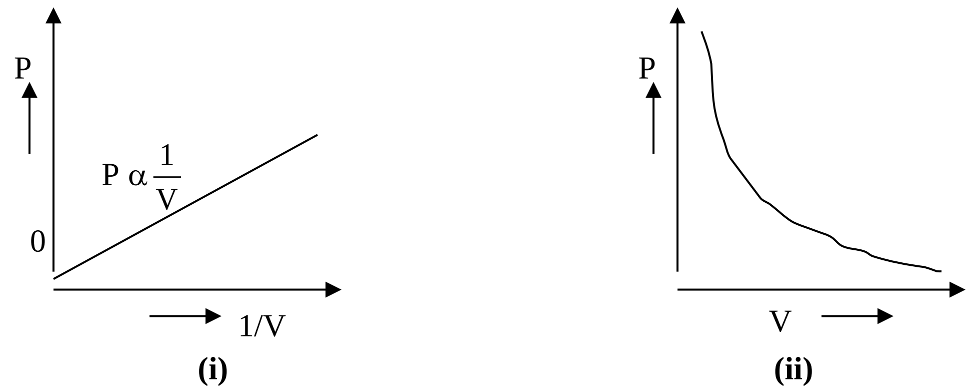


Fig. 1.

Example 1

Given that the volume of air at a pressure of 35 Nm^{-2} is 8.5 m^3. Assuming the temperature is kept constant, calculate its volume when its pressure is 12 Nm^{-2}.

Solution

$P_1 = 35\ Nm^{-2}$ $\quad V_1 = 8.5\ m^3$ $\quad P_2 = 12\ Nm^{-2}$ $\quad V_2 = ?$

$$V_2 = \frac{P_1V_1}{P_2} = 24.8\ m^3$$

Charles's Law

Charles's law deals with the behaviour of a given mass of gas when the pressure (P) and the amount of the gas (n) are kept constant and volume (V) is allowed to vary with temperature (T).

Charles's state that at constant pressure, the volume of a given amount of gas increases by a constant fraction of its volume at 0°C for each Celsius degree rise in temperature.

Mathematically,

$$V \propto T \text{ (at constant n and P)} \qquad (4)$$

If $V_o =$ volume of the gas at 0°C and $V_t =$ volume of the gas at t°C, then γ is expressed as:

$$\gamma = \frac{\Delta V}{V_o \Delta\theta}$$

$$= \frac{V_t - V_o}{V_o (t - 0)}$$

$$\therefore \quad \gamma = \frac{V_t - V_o}{V_o t}$$

$$\therefore \quad V_t = V_o + \gamma V_o t$$

$$V_t = V_o (1 + \gamma t) \qquad (5)$$

Note:

V_0 = volume of the gas at 0°C and

t = actual temperature using the Celsius scale and not for any selected temperature rise.

The value of γ for most gases is $\left(\frac{1}{273}\right)$. Eq. (5) becomes

$$V_t = V_o \left(\frac{273 + t}{273}\right) \qquad (6)$$

But

$$\left.\begin{aligned} (273 + t) &= T \text{ and that} \\ 273 &= T_o \end{aligned}\right\} \qquad (7)$$

Using Eq. (7) in Eq. (6), we obtain

$$\therefore \quad V_t = \frac{V_o T}{T_o}$$

On rearranging the terms, we obtain

$$\left.\begin{aligned} &\therefore \frac{V_t}{T} = \frac{V_o}{T_0} = \text{Constant} \\ &\text{i.e.} \qquad V_t \propto T \end{aligned}\right\} \qquad (8)$$

When the volumes V is plotted on the vertical axis against their corresponding temperature t on the horizontal axis, a linear graph as shown in fig. 2 is obtained.

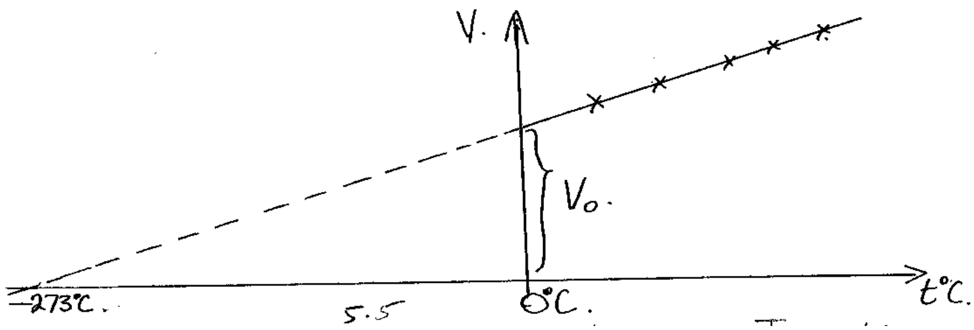


Fig. 2: A graph of Volume versus Temperature

The volume at 0°C, V_o may be extrapolated so that r, the volume coefficient could be determined.

$$\gamma = \frac{V_t - V_o}{V_o \times t}$$

Using the absolute scale temperature, it will be observed from the graph that

$$\frac{V}{T} = \text{Constant.}$$

$$\therefore \quad \frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad (9)$$

Alternatively, Charles's law states that the volume of a given mass of gas is directly proportional to its absolute temperature provided the pressure is kept constant.

Example 2

Some hydrogen gas a volume of 250 cm^3 at 20°C. If the pressure is kept constant, at what temperature will its volume be 100 cm^3?

Solution

$V_1 = 250\ cm^3$, $T_1 = 20°C = (273 + 20)\ K = 293\ K$

$V_2 = 100\ cm^3$ $T_2 = ?$

Using Charles's law

$$\therefore \quad \frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{Constant}$$

$$T_2 = \frac{V_2 T_1}{V_1} = \frac{100\,cm^3 \times 293\,K}{250\,cm^3} = 117.2\ K$$

$T_2 = 273 + t = 117.2\ K$

$\therefore \quad t = 117.2 - 273 =$ **-155.8°C**

Pressure Law

The pressure law states that: **"For a given mass of a gas at constant volume, its pressure increases by a constant fraction of pressure at 0^{o}C for each Celsius degree rise in temperature".**

If P_o is the pressure of the gas at 0^{o}C and P, the pressure at t°C, then the pressure coefficient β is defined as:

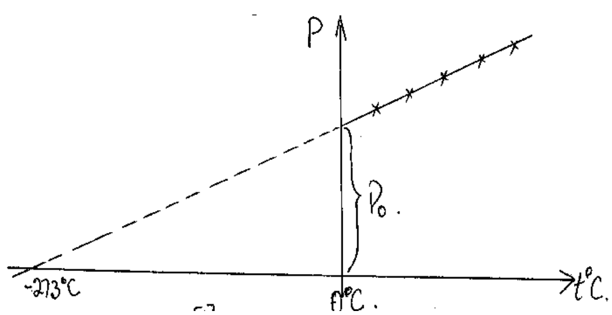
$$\beta = \frac{\Delta P}{P_o t}$$

$$\therefore \quad \beta = \frac{P_t - P_o}{P_o t}$$

$$\therefore \quad P_t - P_o = \beta P_o t$$

$$\therefore \quad P_t = P_o (1 + \beta t) \qquad (9)$$

Graph of Pressure versus Temperature shows that the pressure on the gas varies linearly with the temperature. When the graph is extrapolated, the pressure P_o at 0^{o}C can be read from the graph. Further extrapolation produces the absolute temperature which will be found to be approximately equal to -273^{o}C.



The slope of the graph $\beta = \frac{\Delta P}{P_o t}$

$$\beta = \frac{P_t - P_o}{P_o t}$$

$$\therefore \quad P_t = P_o (1 + \beta t) \qquad \text{[Same as equation (9)]}$$

Put $\beta = \frac{1}{273}$, we obtain

$$P_t = P_o \left(\frac{273 + t}{273} \right) \qquad (10)$$

Using the absolute scale of temperature,

$$273 + t = T$$

$$\therefore \quad P_t = \frac{P_o}{273} \times T = \frac{P_o T}{T_o}$$

$$\therefore \quad \frac{P}{T} = \frac{P_o}{T_o} = K \qquad (11)$$

$$\therefore \quad P_t = KT$$

Hence, $P \ \alpha \ T$ (12)

This implies that the pressure of the gas is directly proportional to its absolute temperature provided the volume is kept constant.

Example 3

At $22°C$ the pressure in gas cylinder is $0.96 \times 10^6 \ Nm^2$. What is the pressure in the cylinder if the cylinder is partly immersed into water at $15°C$?

Solution

$P_1 = 0.96 \times 10^6 \ Nm^2$ $\qquad T_1 = 20^oC = (273 + 22) \ K = 295 \ k$

$P_2 = \ ?$ $\qquad T_2 = 15^oC = (273 + 15) \ K = 288 \ K$

Using the pressure law:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\therefore \quad P_2 = \frac{P_1 T_2}{T_1} = \frac{0.96 \times 10^6 \ Nm^2 \times 288}{295} = \mathbf{0.937 \times 10^6 \ Nm^{-2}}$$

Equation of State for Ideal Gases

In physics, there are two kinds of gases: **real gases and ideal gases.**

The equation of state for ideal gases is obtained by combining either Boyle's law or Charles law or Pressure law

$$\therefore \quad \frac{PV}{T} = \left(\frac{P_o V_o}{T_o}\right) = K = \text{Constant} \qquad (13a)$$

For a given mass of gas $PV = KT$ (13b)

At extremely low pressure, all gases closely obey Boyle's law.

Universal gas Constant

From the Eq. (13b) $PV = KT$

The pressure P_o and 273 (T_o) fix the density of the gas when the volume V_o is proportional to the mass of the gas considered. Therefore, K varies directly as the mass of the gas.

For one mole of a gas, R replaces the constant K.

$$\therefore \quad PV = RT \qquad (14)$$

Equation (14) is the ideal gas equation for one mole of the gas.

Generally, for n mole of a gas we would write:

$$PV = nRT \qquad (15)$$

Where, n = molar fraction (number of moles)

R = Universal gas constant for a mole of a gas.

The number of moles of the gas, $n = \frac{\mathrm{m}}{\mathrm{M}}$ (16)

Where, m = mass of gas in gramme and

M = molecular weight of the gas

$$\therefore \quad PV = \frac{\mathrm{mRT}}{\mathrm{M}} \qquad (17)$$

To obtain the value of Universal Gas Constant, R for 1 mole of a gas recall Eq. (15)

$$PV = nRT$$

$$\therefore \quad R = \frac{\mathrm{PV}}{\mathrm{nT}}$$

Where n = 1,

At S.T.P.

Standard volume, V (volume that one mole of gas occupies) = 22.4 litres.

$= 22.4 \times 10^{-3} m^3 = 22.4\ \mathrm{dm}^3$

Standard Pressure, P = 76 cmHg $= 1.0129 \times 10^5 Nm^2$

Standard temperature, $\mathrm{T} = 273\ \mathrm{K}$

One mole of any gas contains 6.03×10^{23} molecules

This numerically equal to the Avogadro's number, N.

$$R = \frac{1.0129 \times 10^5 \mathrm{Nm}^2 \times 22.4 \times 10^{-3} m^3}{1 \times 273\ \mathrm{K}}$$

$$\mathrm{R} = 8.31\ \mathrm{J.mol^{-1}K^{-1}} \qquad (18)$$

This is the value for the molar gas constant for all gases.

Example 4

350 cm^3 of oxygen gas is collected at temperature $25^{\circ}\mathrm{C}$ and pressure, 725 mmHg. What will be its volume at S.T.P.?

Solution

$P_1 = 725$ mmHg $\quad V_1 = 350\ cm^3 \quad T_1 = 25°C = (273 + 25)\ K = 298\ K$
$P_2 = 760$ mmHg $\quad V_2 = ? \quad T_1 = 0°C = 273\ K$
Using

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore \quad V_2 = \frac{P_1V_1T_2}{P_2T_1} = \frac{725 \times 350 \times 273}{760 \times 298} = \mathbf{305.87\ cm^3}$$

Real Gas Equation

An ideal gas will obey Boyle's law at any temperature. However, real/perfect gases such as air, oxygen, nitrogen etc. will obey Boyle's law within less than one part in a thousand at ordinary pressures and temperatures. At higher pressures and lower temperature, the deviations are more pronounced. In other words, the relation PV is no longer valid.

Kinetic theory of gases suggest that Boyle's law should be obeyed if the molecules are themselves infinitesimally small and if they do not attract each other at all. These assumptions are not true for any real gas. Thus $PV = nRT$ cannot be used for real gases.

In order to account for the difference between the behaviour of a gas and that of an ideal gas, we have to allow for the molecular attractions which converts the pressure P to $(P + \chi)$ and the finite volume occupies by the molecules which reduces the volume V of the gas to $(V - \gamma)$. These corrections therefore enable us to re-express $PV = nRT$ as:

$$(P + \chi)(V - \gamma) = nRT \qquad (19)$$

Where $$\chi = \frac{a}{V^2} \text{ and } \gamma = b \qquad (20)$$

Where, a and b are constants for a unit mass of a gas under consideration.
Substituting Eq. (20) in Eq. (19), we obtain:

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT \qquad (21)$$

This is known as the real equation of state. Otherwise called the Van der Waal's equation of state for real gases.

Assessment Questions

1. State the gas laws.
2. Use suitable graphs to illustrate the various gas laws.
3. State the differences between a real gas and an ideal gas.
4. Obtain the express for the equation of state of an ideal gas.

Assignment 4

1. Differentiate between real and ideal gases.
2. A certain mass of gas having volume $250\ \text{cm}^3$ at 0℃ is heated at constant pressure. What is the volume of the gas at 15℃?
3. Derive gas equation $PV = nRT$
4. A gas at -10℃ was heated to 27℃ till its pressure was raised to 75 cm. find its original pressure if the volume remains constant.

References

Achor, E. E. (2015). *General Physics II (Heat, Light &Sound)*, Nigeria: National Teachers' Institute Kaduna.

Guar R. K. and Gupta S. L. (2001). *Engineering Physics (8th ed.)*. New Delhi: Dhanpat Rai

Michael Nelkon and Philip Parker (1995). *Advanced Level Physics* (5th ed.). London:Heinemann.